Chapter 7 CPM/PERT

PERT (Program Evaluation and Review Technique)

The PERT method for project analysis assumes that three estimates of activity duration follow a beta distribution.

1. Optimistic time a_{ij}
   The shortest possible time required for the completion of activity (i,j)

2. Pessimistic time b_{ij}
   The longest possible time required for the completion of activity (i,j)

3. Most likely time m_{ij}
   The most likely required completing an activity (i,j)

- **Mean of the duration time** $t_{ij} = \dfrac{a_{ij} + 4m_{ij} + b_{ij}}{6}$

  Mean time is used for the activity duration in the network diagram.

- **Variance of the duration time** $v_{ij} = \left(\dfrac{b_{ij} - a_{ij}}{6} \right)^2$

Assumptions:

1. All activity times are assumed to follow beta distribution and statistically independent.
2. The total project time is assumed normally distributed.

Then

E(t) = total project duration time
= $\sum$ (duration time of **critical** activities)

var (t) = var [$\sum$ (duration time of **critical** activities)]
= $\sum$ [var (duration time of **critical** activities)]

Prob (project is finished within X days)
$= P(t \le X)$
$= P(Z \le \dfrac{X - E(t)}{\sqrt{\text{var}(t)}})$ where $Z \sim N(0,1)$

Project Planning and Control

Assume all activities are scheduled to start at their earliest start, the number of men required each week can be calculated by accumulating the resource requirement over all activities taking place in each week.

By using the float of some non-critical activities to delay the start of those activities, it is possible to smooth out the loads so that an even load is possible. ie. a high level of utilization can be achieved by using the minimum resources.

Cost Control

Total variance = Actual expenditure – budgeted expenditure
Operating variance = Value of work done – budgeted expenditure
Efficient variance = Actual expenditure – value of work done

$\Rightarrow$ Total variance = Operating variance + Efficiency variance

Example

Information

Activity	Preceded by	Duration	Men Required	Total Cost
A	–	2 weeks	2	40
B	–	4 weeks	3	80
C	A	4 weeks	2	80
D	B	6 weeks	1	180
E	C, D	4 weeks	3	120

Network

Bar-chart of forward load on resources

Activity Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A2														
B4														
C4														
D6														
E4														
Total														

Bar chart of forward load on cost

Activity Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A2														
B4														
C4														
D6														
E4														
Total														
Acc.Total														

Assume that after 6 weeks activities *A* and *B* was finished, activity *C* was half finished, nothing else had started and the total expenditure to date was 200, we have:

Value of work done =
Budgeted expenditure =
Actual expenditure =
Total variance =
Operating variance =
Efficiency variance =

So the project is behind schedule in terms of physical progress and is over-spent of the work done so far.

Revised network:

Revised Bar-chart of forward load on resources

Activity Time	7	8	9	10	11	12	13	14	15	16
A0										
B0										
C2										
D6										
E4										
Total										

Revised Bar-chart of forward load on cost

Activity Time	7	8	9	10	11	12	13	14	15	16
A0										
B0										
C2										
D6										
E4										
Total										
Acc.Total										

The revised budget for the project would be and would over-run