Chapter 8 CPM/PERT

- For planning, scheduling and control of projects that consist of numerous activities

CPM (Critical Path Method)

2 major components:

Activities: represents the project operations / tasks to be conducted (arrows)

Events: represents project milestone (circles / nodes)

Two or more activities cannot simultaneously share the same start and finish node.
How to solve this problem?

Dummy activities

- A dashed line and an arrowhead to indicate the direction
- Do not consume any time or resource

Be careful that "two or more activities cannot share the same start and finish node" always holds.

Earliest event time (ET): The earliest time by which all activities leading into an event could be completed.

$$ET_j = \max_i \{ET_i + t_{ij}\}$$

Latest event time (LT): The latest time by which all activities leading into an event must be completed if the project is to be completed on time.

$$LT_i = \min_j \{LT_j - t_{ij}\}$$

Project duration: The minimum time required for project completion is equal to the longest time path.

Critical path: The sequence of activities which determines the overall completion time for the project. It can be identified in which the earliest and latest event times are the same.

Slack (float): Activities not on the critical path may be delayed without affecting the overall completion time.

- Total Slack: The time in which the activity's start can be delayed before it becomes critical. $TS_{ij} = LT_j - ET_i - t_{ij}$
- Free Slack: The time in which the activity's start can be delayed without affecting the following activities. $FS_{ij} = ET_j - ET_i - t_{ij}$

Example 1 (95 final)
The management of World Airways wants to determine the minimum amount of time needed to turn around a plane from the moment it reaches the gate until it is ready to leave the gate. To that end, the Flight Manager has identified the following tasks that need to be accomplished between arrival and departure:

Task	Description	Task time (min.)
A	Unload the passengers	15
B	Unload the luggage	25
C	Refuel the engines	30
D	Clean the interior	15
E	Load the meals	15
F	Load the luggage	20
G	Board the passengers	20
H	Perform the safety check	10

The meals cannot be loaded nor the interior cleaned until the passengers are unloaded. The departing luggage cannot be loaded until the arriving luggage has been unloaded. The passengers cannot board until the interior is cleaned. The safety check can be performed only after the engines have been fueled and the meal, luggage, and passengers have been loaded.

a) (2%) Identify the immediate predecessors of each task.

Task	Immediate Predecessors	Task	Immediate Predecessors
A (15)		E (15)	
B (25)		F (20)	
C (30)		G (20)	
D (15)		H (10)	

b) (6%) Draw a network diagram.

c) (4%) Determine the minimum amount of time needed to turn around a plane from the moment it reaches the gate until it is ready to leave the gate. Identify the critical activities.
Minimum amount of time:
Critical activities:

d) (4%) Determine the earliest and latest start and finish times for each activity.

Task	Earliest start	Finish time	Latest start	Finish time
A15				
B25				
C30				
D15				
E15				
F20				
G20				
H10				

e) (4%) Determine the total float and free float for each non-critical activity.

Non-critical task	Total float	Free float
B		
C		
E		
F		

Example 2 (2001 final)

A research institute of the University has acquired a derelict building that is to be refurbished as a computer center. The activities to be carried out are as follows:

(a) Repairs of roof (4 weeks), must be done before any other work can be started.
(b) Plumbing (2 weeks), can be started as soon as the roof is finished.
(c) Structural alterations to interior (4 weeks), can be started as soon as the roof is finished.
(d) Installation of cables for computer network (1 week) can be started as soon as the roof is finished.
(e) Interior decoration (2 weeks), must follow plumbing work.
(f) Restroom renovation (1 week), must follow plumbing work.
(g) Fitting of control area (6 weeks), cannot be done until cables and plumbing are completed.
(h) Exterior landscaping (8 weeks), cannot be done as soon as roof repairs are complete.
(i) Carpets (1 week), after all works (except Furnishing) are completed.
(j) Furnishing (1 week), after all works are completed.

a) What would be the shortest time in which the building could be ready?

Task	Immediate Predecessors	Task	Immediate Predecessors
A (4)	-	F (1)	B
B (2)	A	G (6)	B,D
C (4)	A	H (8)	A
D (1)	A	I (1)	A,B,C,D,E,F,G,H
E (2)	B	J (1)	I

The shortest time in which the building could be ready is

b) What are the critical activities here?

c) Find the total slack time and the free slack time for non-critical activities.

Non-critical task	Total float	Free float
C		
D		
E		
F		