STAT 3001

FOUNDAN OF FIN and MANAGERIAL STA

Tutorial 4

1. Yield to maturity (YTM)

   Yield to maturity approximates the effect of the whole term structure on a bond by a single rate. Denote F_i as cash flow of the bond in each year.

   Annual compounding

$$\text{Price} = \sum_{i=1}^{m} \frac{F_i}{(1 + YTM)^{t_i}}$$

   Continuous compounding

$$\text{Price} = \sum_{i=1}^{m} \lim_{n\to\infty} \frac{F_i}{\left(1 + \frac{YTM^c}{n}\right)^{nt_i}} = \sum_{i=1}^{m} F_i e^{-y^c t_i}$$

   E.g. a 3 year bond (annual compounding) with 4% coupon rate (paying annually) and \$100 face value can be priced by ($y = 5\%$)

   What if the bond is continuous compounding? ($y^c = 5\%$)

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(b) (9 marks) We have two Government bonds. The first bond has maturity of 1 year, paying coupon semi-annually with coupon rate of 20%, and has yield to maturity of 7.8% (continuously compounding convention). The second bond is a zero coupon bond having maturity of 1 year and yield to maturity of 8% (continuously compounding convention). Now the Government wants to issue a new 1-year bond selling at par, paying coupon semi-annually. What is the coupon rate should the Government set?

2. Duration

Macaulay's duration (D) is computed as the weighted average of the times for the payments made by the bond. The weighting of each time is the discounted value of the payment, which shows the importance of that payment. This is just a measure of effective maturity of that bond.

Modified duration (MD) is computed as -1 times the relative sensitivity, the relative sensitivity is calculated as the derivative of the bond price with respect to yield divided by the bond price.

$Duration is defined as absolute sensitivity, which can be calculated by the relative sensitivity times the bond price.

Modified duration and $Duration measures the relative change and absolute change of bond price in response to the small change in yield respectively.

Annual compounding

$$D = \frac{\sum_{i=1}^{m} \frac{t_i F_i}{(1+y)^{t_i}}}{V(y)}$$

$$MD = -\frac{V'(y)}{V(y)} = \frac{\sum_{i=1}^{m} \frac{t_i F_i}{(1+y)^{t_i+1}}}{V(y)} = \frac{D}{1+y}$$

$$\$\text{Duration} = V'(y) = -\sum_{i=1}^{m} \frac{t_i F_i}{(1+y)^{t_i+1}}$$

Continuous compounding

$$D = MD = -\frac{V'(y)}{V(y)} = \frac{\sum_{i=1}^{m} t_i F_i e^{-y^c t_i}}{V(y)}$$

$$\$\text{Duration} = V'(y) = -\sum_{i=1}^{m} t_i F_i e^{-y^c t_i}$$

3. Duration Hedging

Hedge ratio (q) is the number of hedging instrument you have to hold in order to hedge one unit of the original bond portfolio. Denote the bond portfolio as P and the hedging instrument as H.

$$dP + qdH = \left(P'(y) + qH'(y)\right)dy = 0$$

$$q = -\frac{P'(y)}{H'(y)} = \frac{-P \times MD_P}{H \times MD_H}$$

Example

An investor holds 100,000 units of a bond whose features are summarized in the following table. He wishes to be hedged against a rise in interest rates.

Maturity	Coupon rate	YTM	Face value
18 Years	9.5%	8%	$100

Characteristics of the hedging instrument, which is a bond here, are as follow:

Maturity	Coupon rate	YTM	Face value
20 Years	10%	8%	$100

Coupon frequency and compounding frequency are assumed to be annual.

a. Calculate the modified durations for the two bonds.

b. What is the quantity of the hedging instrument that the investor has to sell?

c. Suppose that the YTM increases instantaneously by 0.1%, what happens if the bond is not hedged? What happens if the bond is hedged as above?