STAT 3001

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Tutorial 3

1. Bond Initial Pricing

When talking about interest rate, the most fundamental interest rate product is bond. There are mainly two basic types of bonds, namely coupon bearing bond and zero coupon bond. The names simply implies coupon bearing bond will pay the, so called, coupons which are some intermediate repayments to the holder and zero coupon bond will pay nothing but only the principal.

Calculate the bond price for each coupon bond (semiannually compounding) with face value $100

Time to maturity	Coupon rate	Zero rate	Price
0.5	7.82%	4.59%	
1	6.95%	4.24%	
1.5	6.56%	3.79%	

2. Bootstrapping

In finance, bootstrapping is a method for constructing a yield curve from the prices of a set of bonds.

Example (14 final)

Suppose we know from market prices the following zero rates (semi-annually compounded) with maturities inferior or equal to 1 year:

Maturity (in years)	0.25	0.5	0.75	1
Zero rate (%)	2.50	2.60	2.80	3.00

In addition, we have the following bonds (each with face value \$100, coupons being paid semi-annually) information:

	Maturity (in years)	Coupon rate (%)	Bond price
Bond 1	1.5	3.0	98.3
Bond 2	2.0	3.5	98.7
Bond 3	2.5	4.0	101.6

Compute the zero rates (semi-annually compounded) for the following three maturities: 1.5 years, 2 years, and 2.5 years.

3. Principle Component Analysis and coding

PCA is a statistical technique to find a set of uncorrelated random variables to express most information of a set of highly correlated random variables and achieve dimension reduction.

For a random vector $Y = \begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix}$ with covariance matrix Σ.

First of all, Σ is positivesemidefinite and therefore the following decomposition is guaranteed (This is a result of Linear algebra. You only need to accept it).

$\Sigma = \mathrm{B\Psi B^T}$ with B and Ψ being pxp matrice. The following propertieshold :

1, Ψ is a diagonal matrix with all diagonal elements being nonnegative.

2, The decomposition is arranged such that, if Ψ is denoted as $\begin{pmatrix} \psi_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \psi_p^2 \end{pmatrix}$. $\psi_1^2 \geq \psi_2^2 \geq \ldots \geq \psi_p^2$

3, $B = \begin{pmatrix} b_1 & \cdots & b_p \end{pmatrix}$ with b_i being px1 vectors of unit length for i = 1,..., p. And $b_i{}^T b_j = 0$ for i $\neq$ j.

4, $\mathrm{B^{-1} = B^T}$. That implies $B^T B = I$.

Define $Z = B^T Y$,

$Var(Z) = Var(B^T Y) = B^T Var(Y) B$ (Remark 1)

$= B^T \Sigma B = B^T B \Psi B^T B = \Psi$

Thus <u>the elements of *Z* are uncorrelated</u>.

Another important fact is that

$tr(\Sigma) = tr(B\Psi B^T) = tr(\Psi B^T B)$ (Remark 2)

$= tr(\Psi)$

That illustrate <u>the sum of variance of elements of Y equals the sum of variance of elements of Z</u>.

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1.

(a) (6 marks) Suppose that you have performed PCA on three zero rate data series and obtained the loading matrix B. Please give the missing values (denoted by *) in B:

$$B = \begin{pmatrix} 0.599 & -0.712 & 0.366 \\ * & * & -0.810 \\ 0.550 & * & 0.458 \end{pmatrix}$$