Chapter 2 Investment Appraisal: Basic Methods

- **Net Present Value Rule**
  - Accept investments that have positive NPV
  - Choose the one with the largest NPV

- **Internal Rate of Return (IRR)**
  - IRR is the discount rate which makes NPV = 0.

$$NPV = C_0 + \frac{C_1}{1+IRR} + \frac{C_2}{(1+IRR)^2} + ... + \frac{C_n}{(1+IRR)^n} = 0$$

  - If $C_0 < 0$, we are doing the investment.
    NPV of the project is a smoothly declining function of discounted rate (r).
    Therefore, we will take the project only if r<IRR.

  - If $C_0 > 0$, we are borrowing money from bank.
    NPV of the project is an increasing function of discounted rate (r).
    Therefore, we will take the project only if r>IRR.

Example (04 midterm)
The cash flow projection of a project is given as follows:

Year 0	Year 1	Year 2	Year 3
4000	-3000	-2000	-1000

a. (8%) Use *interpolation* or *extrapolation* to find the (approximate) IRR of your project.
b. (7%) If your cost of capital is 15%, should you undertake the project? Why?

Solution:
a.

b. *Method 1*

Method 2

- **Cycle Problem**
  - For comparing alternatives with different cycle lengths over the same time horizon.

$$PV = PV_{1cycle} + PV\left(\frac{1}{1+r}\right)^k$$

$$PV = \frac{PV_{1cycle}}{1-\left(\frac{1}{1+r}\right)^k}$$, where k is the length of the basic cycle

Example (05 midterm)

You are contemplating the purchase of an air-conditioner and have two choices. Brand A costs HK$8,000 and the estimated expenses on electricity is $600 per month. Brand B costs HK$14,000 and the estimated expenses on electricity is $450 per month. Brand A should have a useful life of 9 years with regular maintenance which costs $450 per year, and Brand B a useful life of 11 years with maintenance cost $500 per year. Which Brand should you buy if the average time of using air-conditioner is three months per year? Will your decision be changed if the average time of using air-conditioner is seven months per year? You may assume that *all utility bills are payable by the end of a year* and the *interest rate is 4% per year*.

Solution:

- **Sensitivity Analysis**
    - Analyse the viability of an investment in relation to each component that is subject to uncertainty
    - Small value of sensitivity
        - → More sensitive
        - → Small change in this item is enough to cause the alter of decision

Example 1 (the sensitivity of C_0)
Find the sensitivity of C_0 for r = 10%

Year	C_0	C_1	C_2	C_3
Cash flow	-2000	1000	1000	1000

Solution:

Example 2 (the sensitivity of sales volumes)
Find the sensitivity of sales volumes
Outlay = \$9000 Sales price = \$20 Unit cost = \$10 Discount rate = 10%
Sales volume: 500 units (1^{st} yr), 400 units (2^{nd} yr), 300 units (3^{rd} yr)

Solution:

- **Single – Investment Risk Analysis**

  Consider an n-year project:

  - Expected return for year t = $\overline{R}_t = \sum_{i=1}^{m} p_{ti} R_{ti}$

    Expected net present values $= E[R_0 + \frac{R_1}{1+r} + \frac{R_2}{(1+r)^2} + \cdots\cdots + \frac{R_n}{(1+r)^n}]$

    $$ENPV = \sum_{t=0}^{n} E\left[\frac{R_t}{(1+r)^t}\right]$$

    $$= \sum_{t=0}^{n} \overline{R}_t (1+r)^{-t}$$

  - Variance of return for year t = $\sigma_t^2 = E(R_t - \overline{R}_t)^2 = \sum_{i=1}^{m} p_{ti} (R_{ti} - \overline{R}_t)^2$

    Variance of the NPV = $\sigma_{NPV}^2 = Var[\sum_{t=0}^{n} R_t (1+r)^{-t}]$

    $$= \sum_{t=0}^{n} \sigma_t^2 (1+r)^{-2t} + \sum_{\substack{s=0 \\ s \neq t}}^{n} \sum_{t=0}^{n} \frac{\operatorname{cov}(R_s, R_t)}{(1+r)^{s+t}}$$

    If returns are independent, cov (R_s , R_t) = 0

    $$\sigma_{NPV}^2 = \sum_{t=0}^{n} \sigma_t^2 (1+r)^{-2t}$$

  - The standard deviation of NPV is used as a measure of total risk for the investment.
  - Risk increases as variance increases.
  - If $NPV \sim N(\mu, \sigma^2)$, it is interested to find $\Pr(NPV < 0)$ using normal approximation.

Example 3
Initial investment = 2000 r = 10%

Year 1	Return	3000	6000
	Prob	0.6	0.4
Year 2	Return	5000	7000
	Prob	0.7	0.3

Find E(NPV) and SD(NPV).

Solution:

Example 4 (04 midterm)
It is estimated that an investment will have an expected return of 5% and a risk of 8%. What is the chance of making a loss for this investment? State your assumption.

Solution:

- **Coefficient of variation**
  - C = $\dfrac{\sigma_{NPV}}{ENPV}$
    If σ_{NPV} is interpreted as risk, then C represents units of risk per unit of return.
  - Smaller value of C is better.
  - Assume NPV is approximately normally distributed, then $\dfrac{\text{NPV} - \text{ENVP}}{\sigma_{\text{NPV}}} \sim \text{N}(0, 1)$
    95% CI for $NPV = (ENPV \pm 1.96\sigma_{NPV})$

Example 5
The senior management of a company decided to accept only projects that carry no more than 5% chance of making a loss. Interpret this policy in terms of coefficient of variation. State your assumption.

Solution:

- **Simulation**
  - Steps are as follows:
    1. Generate set of random variables (involved in the cash flow analysis) from their distributions.
    2. For each set of random variables, compute NPV.
    3. Repeat the process many times; the distribution of NPV can be approximated.