Chapter 2 Investment Appraisal: Basic Methods

- **Discounted Cash Flow (DCF) methods**: discount future cash flow according to their distance from the present and the rate of interest (discount rate).

- **Present Value and Future Value:**
  - PV = the present value of the sum invested at *time zero* (now)
  - FV = the future value of the investment at the *end of the nth year*
  - r = the interest rate / the discount rate as an *annual* percentage
  - n = the no. of *years* of the investment

$$FV = PV(1+r)^n \qquad\qquad PV = FV\frac{1}{(1+r)^n}$$

- **GPV and NPV:**
  - Gross Present Value (GPV) = the sum of the present values of the future cash flows

$$GPV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots\cdots + \frac{C_n}{(1+r)^n}$$

  - Net Present Value (NPV) = $C_0 + GPV$

$$NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots\cdots + \frac{C_n}{(1+r)^n}$$, where C_0 is the initial cash flow

Example (95 midterm)

I can buy a plot of land for \$120,000 and spend a further \$120,000 to build a house on it. I expect to sell it for \$300,000 in a year’s time. A similar risk investment would give me an expected return of 15%.

a) What is the expected revenue from my house-build venture?

b) What is the NPV of the project?

- **Annuities, Deferred Annuities, and Perpetuities:**
  - ❖ Annuities: pays a fixed sum S each year for a specified no. of years, say n years

$$PV = \frac{S}{1+r} + \frac{S}{(1+r)^2} + \cdots\cdots + \frac{S}{(1+r)^n}$$
$$= S\ \frac{1-(1+r)^{-n}}{r}$$
$$= S\ a(r,n)$$

where $a(r,n)$ is called **annuity factor**

  - ❖ Perpetuities: a special case of an annuity where a contract runs indefinitely and there is no end to the payments

$$PV = S \lim_{n\to\infty} \left| \frac{1-(1+r)^{-n}}{r} \right| = \frac{S}{r}$$

  - ❖ Deferred Annuity: a kind of annuity which payment is delayed for some years

    Two Methods ►Differencing Method
    ►Discounting Method

Example

For r = 10%

Year	0	1	2	3	4	5	6	7
5 yr annuity		100	100	100	100	100		
Perpetuities		100	100	100	100	100	100	……
Deferred annuity				100	100	100		

Solution:

a) PV of 5 yrs annuity:

b) PV of perpetuities:

c) Deferred annuity :
(by differencing method)

(by discounting method)

Example

One will receive from his late uncle's estate $50 in 1 year's time and annually thereafter in perpetuity. What is the value of this perpetuity at an interest rate of 10 %? How much is this perpetuity worth if it begins in 10 years time instead of 1 year time?

Solution:

Following the formula of perpetuity, we have

Then we use relationship between the PV and FV, we get

- **Effective Interest Rate:**
  - The effective interest rate is the number r_e such that it has the effect of compounding on yearly growth:

    $$1 + r_e = (1 + \frac{r}{m})^m \quad \text{or} \quad r_e = (1 + \frac{r}{m})^m - 1$$

    where r is the nominal rate (basic yearly rate).

- **Applications:** (1) Mortgage repayments
  (2) Hire purchase
  (3) Tax loan plan

Example on Mortgage repayments

The mortgage loan agreement between Mr. Chan and a local bank is given as follows:

a) Loan amount P = HK$ 1,500,000
b) Interest rate r = 3.25% per annum
c) Total number of monthly installments **n = 240**

1. Find the monthly repayment amount I.
2. Express I as a function of P, r and n.

Solution:

1.

2.

Example on Hire purchase

The cost of an equipment is $2000, and the period of the hire purchase agreement is 3 years. The quoted interest rate is 10%. Find the monthly rate.

Solution:

Total interest charged =

Monthly payments =

Example on Tax loan plan (00 midterm)

p:	loan amount	h:	handling charge
r_m:	*monthly* compound rate	r_e:	effective annual interest rate
i:	flat rate	n:	number of installments

A local bank offers an interest rate of 0.65% per month flat in a tax loan plan with handling charge 1%.

a. Find the monthly repayment amount with 12 installments for a loan amount of HK $80000. (5%)

b. Find also the annualized percentage rate of this tax loan plan. (15%)

Solution:

a. The total amount to be charged for interest is:

The total interest is:

Therefore, the monthly installment is:

b. Because Loan amount = monthly installment × $a(r_m, n)$, we have